D 73288 (Pages : 3)

Name..

Reg. No.....................................

FIRST SEMESTER B.A./B.Sc. DEGREE EXAMINATION, NOVEMBER 2019

(CBCSS—UG)

Mathematics

MTS 1C 01—MATHEMATICS—I

(2019 Admissions)

Time : Two Hours   Maximum : 60 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Maximum Marks 20.

1. Find the derivative of $f(x) = 3x^2 + 8x$ at $x_0 = -2$ and $x_0 = \frac{1}{2}$.

2. A rock thrown down from a bridge has fallen $4t + 4.9t^2$ meter after t seconds. Find its velocity at $t = 3$.

3. Find $\lim_{x \to \infty} \frac{5x^2 - 3x + 2}{x^2 + 1}$.

4. Suppose that $f(t) = \frac{1}{4}t^2 - t + 2$ denotes the position of a bus at time t. Find the acceleration.

5. A bagel factory produces $30x - 2x^2 - 2$ dollars worth of bagels for each x worker hours of labour. Find the marginal productivity when 5 worker hours are employed.

6. The velocity of a particle moving along a line is $3t + 5$ at time t. At time 1, the particle is at position 4. Where is at time 10 ?

7. Use the second derivative test to analyze the critical points of the function $f(x) = x^3 - 6x^2 + 10$.

Turn over

8. Find inflection point of the function $f(x) = x^2 + \frac{1}{x}$.

9. Find $\lim_{x \to 0^+} x \ln x$.

10. Draw the graph of the step function g on $[0,1]$ defined by $g(x) = \begin{cases} -2, \text{ if } 0 \le x < \frac{1}{3} \\ 3, \text{ if } \frac{1}{3} \le x \le \frac{3}{4} \\ 1, \text{ if } \frac{3}{4} < x \le 1 \end{cases}$. Compute the signed area of the region between its graph and the x-axis.

11. Find the sum of the first n integers.

12. Find $\int_0^4 \left(t^2 + 3t^{\frac{7}{2}}\right) dt$.

Section B

Answer any number of questions.
Each question carries 5 marks.
Maximum Marks 30.

13. (a) Differentiate $\frac{1}{\left(x^3+3\right)\left(x^2+4\right)}$.

(b) Calculate approximate value for $\sqrt{8}$ using the linear approximation around $x_0 = 9$.

14. Find the equation of the tangent line to the curve $2x^6 + y^4 = 9xy$ at the point (1, 2).

15. Water is flowing into a tub at $3t + \frac{1}{(t+1)^2}$ gallons per minute after t minutes. How much water is in the tub after 2 minutes if it started out empty.

16. State mean value theorem. Let $f(x) = \sqrt{x^3 - 8}$. Show that somewhere between 2 and 3 the tangent line to graph of f has slope $\sqrt{19}$.

17. Find the dimensions of a box of minimum cost if the manufacturing costs are 10 cents per square meter on the bottom, 5 cents per square meter on the sides, and 7 cents per square meter on the top. The volume is to be 2 cubic metres and height is to be 1 metre.

18. The region between the graph of x^2 on $[0, 1]$ is revolved about the x-axis. Sketch the resulting solid and find its volume.

19. Find the area between the graphs of $y = x^3$ and $y = 3x^2 - 2x$ between $x = 0$ and $x = 2$.

Section C

Answer any **one** *question.*

Each question carries 10 *marks.*

Maximum Marks 10.

20. (a) Differentiate $\dfrac{x^{\frac{1}{2}} + x^{\frac{3}{2}}}{x^{\frac{3}{2}} + 1}$.

(b) Find inflection point of the function $f(x) = x^2 + \dfrac{1}{x}$.

21. (a) Find $\lim_{x \to 0} \left(\dfrac{1}{x \sin x} - \dfrac{1}{x^2} \right)$.

(b) Find average value of $f(x) = x^2 \sin x^3$ on $[0, \pi]$.

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FIRST SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2017

(CUCBCSS—UG)

Complementary Course

MAT 1C 01—MATHEMATICS

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer all **twelve** *questions.*

1. At what points are function $f(x) = \frac{1}{(x+2)^2} + 4$ continuous ?

2. Define critical point of a function.

3. Suppose $\lim_{x \to c} f(x) = 5$ and $\lim_{x \to c} g(x) = -2$. Find $\lim_{x \to c} f(x)\, g(x)$.

4. Find the norm of the partition $[0, 1.2, 1.5, 2.3, 2.6, 3]$.

5. Find absolute minima of $y = x^2$ on $(0, 2]$.

6. Find the interval in which $y = x^3$ is concave up.

7. $\frac{d}{dx} \int_a^x f(t)\, dt =$ ————————.

8. Find dy if $y = x^5 + 37x$.

9. Define average value of a function f on $[a, b]$.

10. Find $\lim_{x \to -\infty} \frac{\pi\sqrt{3}}{x^2}$.

11. Define horizontal asymptote of the graph of a function.

12. Find $\lim_{x \to 2} \frac{3-x}{3+x}$.

(12 × 1 = 12 marks)

Turn over

Part B (Short Answer Type)

Answer any **nine** *questions.*

13. If $2 - x^2 \le g(x) \le 2\cos x$ for all x, find $\lim_{x \to 0} g(x)$.

14. If $\lim_{x \to 4} \frac{f(x) - 5}{x - 2} = 1$, find $\lim_{x \to 4} f(x)$.

15. Find the derivative of $y = \sqrt{x}$ for $x > 0$. Find the tangent line to the curve $y = \sqrt{x}$ at $x = 4$.

16. Area A of a circle is related to its diameter by the equation $A = \frac{\pi}{4} D^2$. How fast is the area changing with respect to the diameter when the diameter is 10 m ?

17. Find absolute extreme values of $g(t) = 8t - t^4$ on $[-2, 1]$.

18. Show that $\lim_{x \to -\infty} \frac{1}{x} = 0$.

19. The radius r of a circle increases from $r_0 = 10\ m$ to 10.1 m. Estimate the increase in the circle's area A by calculating dA. Compare this with true change ΔA.

20. Find a lower bound for the value of $\int_0^1 \cos x\, dx$ using the inequality $\cos x \ge 1 - x^2/2$.

21. Use Max-Min inequality to find upper and lower bounds for the value of $\int_0^1 \frac{1}{1 + x^2}\, dx$.

22. Find the area of the region between $y = 4 - x^2, 0 \le x \le 3$ and the x-axis.

23. Find the function with derivative $f'(x) = 2x - 1$ passing through the point $P(0, 0)$.

24. Find $\frac{d}{dx} \int_0^{t^4} \sqrt{u}\, du$.

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions.*

25. Find the slope of the curve $y = 1/x$ at $x = a$. Where does the slope equal $-1/4$? What happens to the tangent to the curve at the point $(a, 1/a)$ as a changes ?

26. Show that functions with zero derivatives are constant.

27. Find the asymptotes of the graph of $f(x) = \dfrac{-8}{x^2 - 4}$.

28. Find $\lim_{x \to 0^+} \dfrac{\sqrt{h^2 + 4h + 5} - \sqrt{5}}{h}$.

29. Show that functions with the same derivative differ by a constant.

30. Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}, 1 \le x \le 2$ about the x-axis.

31. Express the solution of the initial value problem $\dfrac{ds}{dt} = f(t), s(t_0) = s_0$ in terms of integral.

32. Show that if f is continuous on $[a, b], a \ne b$ and if $\int_a^b f(x)\,dx = 0$, then $f(x) = 0$ at least once in $[a, b]$.

33. Show that if f has a derivative at $x = a$ then f is continuous at a.

(6 × 5 = 30 marks)

Part D (Essay Type)

Answer any **two** *questions.*

34. Find the intervals on which $g(x) = -x^3 + 12x + 5, -3 \le x \le 3$ is increasing and decreasing. What are the critical points ? When does the function assume extreme values and what are these values ?

35. Find the volume of the solid generated by revolving the regions bounded by the curve $x = \sqrt{5}y^3, x = 0, y = -1, y = 1$ about x-axis.

Turn over

36. Let $f(x)=\begin{cases} 3-x, & x<2; \\ 2, & x=2; \\ \dfrac{x}{2}+1, & x>2 \end{cases}$

(a) Find $\lim_{x\to 2^+} f(x)$ and $\lim_{x\to 2^-} f(x)$. and $f(2)$.

(b) Does $\lim_{x\to 2} f(x)$ exist ? If so, what is it ? If not, why not?

(c) Find $\lim_{x\to -2^+} f(x)$ and $\lim_{x\to -2^-} f(x)$.

(d) Does $\lim_{x\to -2} f(x)$ exist ? If so, what is it ? If not, why not ?

(2 × 10 = 20 marks)